PERMISSIBLE HEAT LOADS ON COOLED LASER MIRRORS UNDER NONUNIFORM IRRADIATION

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The results of a study are presented pertaining to a theoretical analysis of the temperature field and the field of thermal strains and stresses in cooled laser mirrors produced by nonuniform heating. The limiting thermal fluxes have also been calculated.

The basic requirements for mirrors of power laser systems are a high threshold of optical breakdown and small thermal strains. Under steady-state conditions the temperature of these mirrors must be lowered by means of a coolant and held constant by special methods. Several proposals have been made for designing a cooled laser mirror, among them channel cooling [1-3], cooled porous substrate [4], and a mirror in the form of a heat pipe [5]. The density distribution of luminous energy over the mirror surface is usually nonuniform, with maximum density at the center and minimum density at the periphery. Furthermore, several laser beams may impinge on the surface of one mirror. The temperature field as well as the field of thermal strains and stresses have a rather intricate pattern under these conditions. The object of this study will be to estimate the maximum thermal stresses and strains along with the corresponding permissible thermal fluxes, depending on the degree of mirror irradiation nonuniformity and on the mirror cooling intensity.

As basis for the calculation model serves the mirror [5, 7] designed in the form of a circular disk with radius b and thickness δ, rigidly clamped around the perimeter (Fig. 1) or freely supported at the contour. The plane $z = 0$ is reflecting. The plane $z = \delta$ is cooled with an intensity corresponding to a heat-transfer coefficient α. The mirror is heated over a spot with radius $a \ll b$ (a being the radius of the laser beam) by thermal flux of constant density $Q/\pi a^2$. The mirror surface outside the irradiated spot ($a < r \ll b$) and the lateral surface are thermally insulated. For such a configuration and with constant thermophysical properties of the mirror material one can solve the Neumann problem of the steady-state temperature field exactly [8, 9], namely

$$T(\rho,\ \xi) = T_L + \Delta T\left\{\frac{1+\mathrm{Bi}}{\mathrm{Bi}} - \frac{\xi}{\tau} + \frac{2}{\tau\eta}\sum_{k=1}^{\infty}\frac{J_1(\mu_k\eta)\,J_0(\mu_k\rho)}{\mu_k^2 J_0^2(\mu_k)}\,[\varepsilon_k\,\mathrm{ch}(\mu_k\xi) - \mathrm{sh}(\mu_k\xi)]\right\}. \quad (1)$$

Here $\varepsilon_k = [\mathrm{Bi}\ \mathrm{th}(\mu_k\tau) + \mu_k\tau]/[\mathrm{Bi} + \mu_k\tau\ \mathrm{th}(\mu_k\tau)]$; $\Delta T = (Q/\pi b^2)(\delta/\lambda)$ is the temperature drop across the mirror disk under uniform heating ($a = b$), and μ_k are roots of the equation $J_1(\mu_k) = 0$. The temperature of the mirror will be henceforth read relative to the temperature of the liquid coolant, assuming that the latter remains constant and equal to zero. As the initial unstressed state of the mirror will be regarded its state at the temperature of the coolant.

Under uniform heating ($a = b$) the ratio of the temperature drop ΔT across the wall to temperature drop $Q/\pi b^2\alpha$ across the boundary layer of liquid boundary layer is exactly equal to the Biot number. Under nonuniform heating ($a < b$), on the other hand, a large Biot number ($\mathrm{Bi} \gg 1$) corresponds to a boundary condition of the first kind and a small Biot number ($\mathrm{Bi} \ll 1$) corresponds to a boundary condition of the second kind at the cooled surface.

The thickness of the mirror is usually much smaller than its radius ($\delta/b \ll 1$). For calculating the strains (deflection, curvature) and the stresses one can, therefore, use data based on the theory of thermoelasticity for thin circular plates [10]. It can be demonstrated in our case that the normal (in the z-direction) flexural strain $\omega(r)$ of the reflecting surface is [10]

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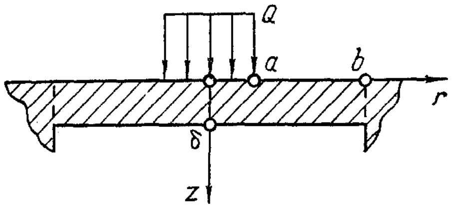

Fig. 1. Schematic diagram of cooled mirror with nonuniform irradiation.

$$\omega(r) = \frac{12(1+\nu)\beta}{\delta^3}\left[\int_r^b \frac{F(r')}{r'}dr' + \varkappa\frac{F(b)}{2}\left(1-\frac{r^2}{b^2}\right)\right], \tag{2}$$

with $F(r) = \int_0^r r'\left[\int_0^\delta T(r', z)\left(z-\frac{\delta}{2}\right)dz\right]dr'$; $\varkappa = -1$ for a plate clamped at the contour, and $\varkappa = (1-\nu)/(1+\nu)$ for a plate freely supported at the contour.

It follows from relation (2) that there are two situations in which a circular plate clamped at the contour will not deform ($\omega = 0$): 1) when the temperature of the plate varies only in the z-direction (uniform heating) and 2) when the temperature of the plate varies only radially. The latter situation is characteristic of nonuniform heating with Bi << 1 and a << b.

According to the theory of thermoelasticity for thin plates [10], only tensile–compressive stresses σ_{rr} and $\sigma_{\varphi\varphi}$ in the plane of the plate are significant (not zero). These stresses are related to the temperature field T(r, z) and to components u_{rr}, $u_{\varphi\varphi}$ of the strain tensor, which depend on the normal strain $\omega(r)$, in accordance with the equations

$$\begin{aligned}\sigma_{rr} &= \frac{E}{1-\nu^2}\left[-(1+\nu)\beta T + (u_{rr} + \nu u_{\varphi\varphi})\right],\\ \sigma_{\varphi\varphi} &= \frac{E}{1-\nu^2}\left[-(1+\nu)\beta T + (u_{\varphi\varphi} + \nu u_{rr})\right],\end{aligned} \tag{3}$$

$$u_{rr} = -\left(z-\frac{\delta}{2}\right)\frac{\partial^2\omega}{\partial r^2};\quad u_{\varphi\varphi} = -\frac{z-\delta/2}{r}\frac{\partial\omega}{\partial r}. \tag{4}$$

It follows from Eqs. (3) and (4) that the internal stresses in a mirror consist of two parts: compressive thermoelastic stresses without flexure of the plate (first terms inside square brackets in Eqs. (3)) and flexural stresses alone (second terms inside square brackets in Eqs. (3)). The second stresses are maximum at both reflecting and cooled surfaces of the mirror, being positive (tensile) at the former (z = 0, r = 0) and negative (compressive at the latter (z = δ, r = 0)). Accordingly, bending or buckling of the mirror lowers the compressive stress at its reflecting surface somewhat. The thinner the mirror is, the stronger is this stress reducing effect. Decreasing the mirror thickness below a certain threshold will, however, result in a prohibitively high flexural strain of the reflecting surface.

Inserting expression (1) for the temperature field into Eqs. (2)-(4), we obtain the sought displacement field of the reflecting surface and the field of internal stresses as functions of the absorbed thermal power Q, of the cooling intensity Bi, of the relative dimension of the irradiated spot η, of the geometrical dimensions of the mirror (δ, b), and of the physical properties of the mirror material

$$\frac{\omega(\rho)}{b} = -(1+\nu)\beta\Delta T\sum_{k=1}^{\infty} D_k\left[J_0(\mu_k) - J_0(\mu_k\rho)\right], \tag{5}$$

$$\sigma_{rr}(\rho, \xi) = -\frac{E\beta\Delta T}{1-\nu}\left\{\frac{T(\rho, \xi)}{\Delta T} + \left(\xi-\frac{\tau}{2}\right)\sum_{k=1}^{\infty} D_k\mu_k^2\left[J_0(\mu_k\rho) - \frac{1-\nu}{\mu_k\rho}J_1(\mu_k\rho)\right]\right\}, \tag{6}$$

$$\sigma_{\varphi\varphi}(\rho, \xi) = -\frac{E\beta\Delta T}{1-\nu}\left\{\frac{T(\rho, \xi)}{\Delta T} + \left(\xi-\frac{\tau}{2}\right)\sum_{k=1}^{\infty} D_k\mu_k^2\left[\nu J_0(\mu_k\rho) + \frac{1-\nu}{\mu_k\rho}J_1(\mu_k\rho)\right]\right\}. \tag{7}$$

TABLE 1. Values of Dimensionless Deflection $\omega(0)/\omega_*$ of Reflecting Surface of Cooled Mirror Clamped at Contour, for $\omega_* = (1 + \nu)\beta\Delta T\delta$

Bi	τ = 0,01			τ = 0,05			τ = 0,1		
	η = 0,1	0,4	0,7	0,1	0,4	0,7	0,1	0,4	0,7
0,01	4980	1041	232	252	139	42,9	55,1	23,9	9,4
0,10	10500	4150	1600	334	156	61,8	66,0	31,1	12,4
1,00	11300	4470	1740	404	178	69,6	84,5	40,9	16,2
10,00	11300	4490	1740	426	181	70,5	92,2	43,3	17,0

Here

$$D_k = \frac{24}{\eta\tau^4}\frac{J_1(\mu_k\eta)}{\mu_k^6 J_0^2(\mu_k)}\left[\left(\varepsilon_k - \frac{\mu_k\tau}{2}\right) + \left(1 + \frac{\varepsilon_k\mu_k\tau}{2}\right)\operatorname{sh}(\mu_k\tau) - \left(\varepsilon_k + \frac{\mu_k\tau}{2}\right)\operatorname{ch}(\mu_k\tau)\right]. \quad (8)$$

It follows from expressions (5)-(8) that the maximum strains and stresses occur at the center of the mirror ($\xi = 0$, $\rho = 0$). When the mirror is uniformly irradiated ($a = b$), then its temperature field is linear in the z-direction. In this case a mirror clamped at the contour will not deform ($D_k = 0$, $\omega = 0$) and the compressive stresses at its reflecting surface will be

$$\sigma_{rr} = \sigma_{\varphi\varphi} = \frac{E\beta\Delta T}{1-\nu}\frac{1+\mathrm{Bi}}{\mathrm{Bi}}. \quad (9)$$

In a mirror which is not clamped, on the contrary, there will be no stresses and the maximum deflection will be

$$\omega(0) = \frac{\beta}{2\pi\lambda}Q \quad (10)$$

regardless of the mirror dimensions [11]. In the case of nonuniform irradiation and Bi $\gtrsim$ 1 the relation (2) yields directly approximate expressions for the flexural strain of a clamped mirror

$$\omega(0) = \frac{\beta Q}{2\pi\lambda}(1+\nu)\ln\frac{b}{a} \quad (11)$$

and for a mirror without clamping

$$\omega(0) = \frac{\beta Q}{2\pi\lambda}\left[1 + (1+\nu)\ln\frac{b}{a}\right]. \quad (12)$$

The data in Table 1 represent the numerically evaluated dimensionless deflection $\Omega = \omega(0)/[(1+\nu)\beta\Delta T\delta]$ at the center of mirrors of various thicknesses with irradiation spots of various dimensions, for various values of the Biot number. It follows from these data that the dimensionless deflection decreases fast with increasing mirror thickness (τ) and with increasing size of the irradiated spot (η). The strain of the mirror decreases also with decreasing value of the Biot number (heat-transfer coefficient). This nontrivial result can be explained by the fact that the temperature of the mirror varies more appreciably in the radial direction than in the axial z-direction when Bi < 1 and a < b but more appreciably in the axial z-direction than in the radial direction when Bi >> 1.

The graph in Fig. 2 depicts the dependence of the surface deflection of a copper mirror on the relative dimensions of the irradiated spot, at a constant power density (0.5 MW/m^2) of absorbed thermal flux within the spot, for various values of the Biot number ranging from 0.01 to 10. The mirror dimensions are here [5] b = 25.4 mm and δ = 1.84 mm. According to the graph in Fig. 2 and expressions (11) and (12), the deflection of the reflecting surface under $Q/\pi a^2$ = const (operation with diaphragm) is maximum when $\eta \simeq 0.6$ at any value of the Biot number. The increase of the strain as the irradiated spot becomes larger within the $\eta < 0.6$ range is related to the attendant increase of absorbed power and the decrease of the strain as the irradiated spot becomes larger within the $\eta > 0.6$ range is related to the attendant linearization of the temperature profile over the mirror thickness, a linear temperature profile being characterized by an absence of strains in a plate clamped at the contour.

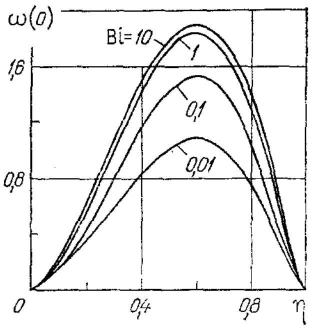

Fig. 2

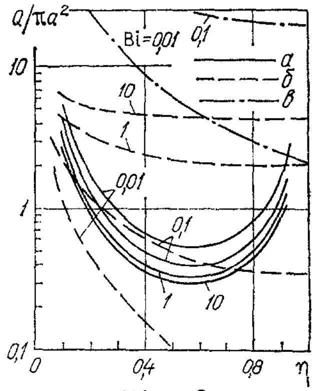

Fig. 3

Fig. 2. Dependence of strain (ω, μm) of reflecting surface on relative dimension ($\eta = a/b$) of irradiated spot, for mirror of relative thickness $\delta/b = 0.07$ clamped at contour under constant heat load $Q/\pi a^2 = 0.5$ MW/m², at various values of Biot number.

Fig. 3. Dependence of maximum permissible thermal flux density ($Q/\pi a^2$, MW/m²) for copper mirror on relative dimension ($\eta = a/b$) of irradiated spot and on Biot number (Bi). a) strain limitation to $\omega(0) \leqslant 1$ μm, b) stress limitation to $\sigma_{rr}(0, 0) \leqslant 0.6 \cdot 10^8$ N/m², c) temperature limitation to $T(0, 0) \leqslant 1080°C$; relative thickness of mirror clamped at contour $\delta/b = 0.07$.

In another study [5] are given experimental data on the strain (approximately 0.4 μm) of a copper mirror of approximately the same dimensions under heating with a thermal power of 107 W over a spot having an area of 2 cm². Unfortunately, neither had the heat-transfer coefficient been determined nor is it indicated whether the mirror had reinforcing ribs on the cooled surface. All this makes it difficult to compare the results of our calculations with the results of study [5].

The graphs in Fig. 3 depicts the dependence of the limiting density $Q/\pi a^2$ of absorbed thermal flux on the relative dimension of the irradiated spot, the limiting thermal flux density being that which will produce any one of the three intolerable effects: 1) melting at the center when $T(0, 0) = T_m$; 2) appearance of irreversible (plastic) strains when the stress at the center exceeds the elastic limit $[\sigma]$ or yield strength; 3) deflection of the reflecting surface by a prohibitive amount $[\omega]$ usually defined in fractions of the laser radiation wavelength such as 1/10 [2-6], for instance, which corresponds to $\omega \simeq 1$ μm in the case of a CO_2 laser. Our calculations are based on $\lambda = 380$ W/(m•K), $E = 1.3 \cdot 10^{11}$ N/m², $[\sigma] = 0.6 \cdot 10^8$ N/m², $\nu = 1/3$, $\beta = 1.7 \cdot 10^{-5}$ K⁻¹, $T_m = 1080°C$, $b = 25.4$ mm, and $\delta = 1.84$ mm. The graph indicates that, as the irradiated part of the copper mirror becomes larger, plastic strain and melting of the mirror will occur at a, respectively, lower thermal flux density (within the spot). The range of thermoelastic strain is, moreover, bounded from above by much lower (almost two orders of magnitude) than those at which the mirror can melt. This upper boundary of the range of elastic strains also depends strongly on the Biot number (cooling intensity).

The limitation on heat loads when $Bi > 0.1$ and $\eta \simeq 0.3$-0.8 are dictated by the need to avoid prohibitive deflection of the reflecting surface (1 μm in the given case). As the relative spot dimension η changes, the heat load which causes the center of the mirror to shift through a certain distance passes through a minimum located within the $\eta = 0.5$-0.7 range (Fig. 2). The dependence on the Biot number is much weaker here than in the cases stress limitation and temperature limitation. The limitation on deflection of the reflecting surface, defined in fraction of the laser radiation wavelength, can now become the governing factor which will limit the heat load on mirrors for lasers operating at wavelength shorter than in the given example.

To the aforementioned factors limiting the heat load on a mirror during steady-state operation must be added still another one, namely the heat-dissipating capacity of the coolant.

NOTATION

z, r, φ, cylindrical coordinates; b, mirror radius, m; δ, mirror thickness, m; a, radius of the impinging laser beam, m; $\xi = z/b$ and $\rho = r/b$, dimensionless coordinates; $\tau = \delta/b$, dimensionless mirror thickness; $\eta = a/b$, relative radius of the irradiated spot; T, temperature, °K; α, heat-transfer coefficient, $W/(m^2 \cdot °C)$; λ, thermal conductivity of the mirror material, $W/(m \cdot °C)$; $Bi = \alpha\delta/\lambda$, Biot number; Q, power absorbed by the mirror, W; β, coefficient of thermal expansion of the mirror material, K^{-1}; ν, Poisson ratio; E, Young's modulus, N/m^2; σ, stress, N/m^2; u_{xy}, components of the strain tensor; $\omega(x)$, deflection, m; $J_n(x)$, Bessel function of first kind of n-th order; and sinh, cosh, tanh, hyperbolic sine, cosine, tangent.

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